

Light Ξ hypernuclei

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Abstract

Arguments in favor of a light ΞNN hypernucleus with $(I)J^P = (3/2)1/2^+$ are presented, within the uncertainties of our knowledge of the baryon-baryon strangeness -2 interactions. If bound, this ΞNN state, being decoupled from the lowest $N\Lambda\Lambda$ system, would be stable. It will also benefit from additional binding due to the electromagnetic interaction what makes it worthwhile to look for. We show how the equivalent state with $J = 3/2$ could never be bound in spite of the attractive interaction of the two-body subsystems. We illustrate our discussion with a full-fledged Faddeev calculation of the ΞNN system using simple potentials that mimic more elaborate interactions. We also make contact with different recent phenomenological interactions from the literature, like the ESC08 Nijmegen potential or quark-model based potentials.

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The physics of hypernuclei is progressing dramatically in both theory and experiment. Theoretically there have been recent proposals of the stability of ${}^4_{\Lambda\Lambda}\text{n}$ [1], the existence of Ξ hypernuclei [2–4], or the strangeness -2 hypertriton [5]. Obviously, all these predictions are subject to the uncertainties of our knowledge of the baryon-baryon interaction, in particular in the strangeness -2 sector. Experimentally, it has been recently reported an emulsion event providing clear evidence of a deeply bound state of the $\Xi^- - {}^{14}\text{N}$ system [6]. A thorough discussion of the present status of the experimental progress in hypernuclear physics can be found in Ref. [7]. To encourage new experiments seeking hypernuclei, it is essential to make a detailed theoretical investigation of the possible existence of bound states, despite some uncertainty in contemporary interaction models [3]. To advance in the knowledge of the details of the $N\Xi$ interaction, high-resolution spectroscopy of Ξ hypernuclei using ${}^{12}\text{C}$ targets in (K^-, K^+) reactions has been awaited [8, 9] and it is now planned at J-PARC [10]. Identification of hypernuclei in coming experiments at J-PARC will contribute significantly to understand nuclear structure and baryon-baryon interactions in the strangeness -2 sector.

When a two-baryon interaction is attractive, if the system is merged with nuclear matter and the Pauli principle does not impose severe restrictions, the attraction may be reinforced. Simple examples of the effect of a third or a fourth baryon in two-baryon systems could be given. The deuteron, $(I)J^P = (0)1^+$, is bound by 2.225 MeV, while the triton, $(I)J^P = (1/2)1/2^+$, is bound by 8.480 MeV, and the α particle, $(I)J^P = (0)0^+$, is bound by 28.295 MeV. The binding per nucleon B/A increases from 1 : 3 : 7. A similar argument could be employed for strangeness -1 systems. Whereas there is no evidence for dibaryon states, the hypertriton ${}^3_{\Lambda}\text{H}$, $(I)J^P = (0)1/2^+$, is bound with a separation energy of 130 ± 50 keV, and the ${}^4_{\Lambda}\text{H}$, $(I)J^P = (0)0^+$, is bound with a separation energy of 2.04 ± 0.04 MeV. This cooperative effect of the attraction in the two-body subsystems when merged in few-baryon states was also made evident in the prediction of a ΣNN quasibound state in the $(I)J^P = (1)1/2^+$ channel very near threshold [11]. Such ΣNN quasibound state has been recently suggested in ${}^3\text{He}(K^-, \pi^\mp)$ reactions at 600 MeV/c [12]. One should also bear in mind how delicate is the few-body problem in the regime of weak binding, as demonstrated in Ref. [13] for the ${}^4_{\Lambda\Lambda}\text{H}$ system.

It is the purpose of this paper to highlight a particular set of quantum numbers in the three-baryon strangeness -2 system that brings together all the expected characteristics as to be bound in nature, this is the $(I)J^P = (3/2)1/2^+$ state. This set of quantum numbers

could be achieved by means of a ΞNN state, being decoupled from the lowest $N\Lambda\Lambda$ state, and it would therefore be stable. It will get two different contributions from the two-body subsystems: the $(i, j) = (1, 0)$ NN state with a spectator Ξ , that will benefit from maximum coupling in isospin space preserving the attraction of the NN subsystem; and the $(i, j) = (1, 0)$ and $(1, 1)$ $N\Xi$ states with a spectator N , that also benefit from maximum coupling in isospin space preserving the expected attractive character of the $N\Xi$ interaction in isospin 1 partial waves [4, 14–17]. Besides, this state will also gain additional binding due to the electromagnetic interaction what makes it worthwhile to look for.

The first evidence of a deeply bound state of $\Xi^- - {}^{14}\text{N}$ has been recently reported [6], indicating that Ξ -nucleus interactions are attractive. Together with other indications of certain emulsion data, these data suggest that the average $N\Xi$ interaction should be attractive [14–16]. In particular, the ESC08c Nijmegen potential for baryon-baryon channels with total strangeness -2 predicted an important attraction in the $i = 1$ $N\Xi$ interaction, with a bound state of 8.3 MeV in the $(i, j) = (1, 1)$ $N\Xi$ channel [14]. The recent update of the ESC08c potential to take into account the new experimental information of Ref. [6] concludes the existence of a bound state in the $(i, j) = (1, 1)$ $N\Xi$ channel with a binding energy of 1.56 MeV [15]. The attractive character of the $i = 1$ $N\Xi$ interaction has also been noticed in the quark-model analysis of Ref. [17], with a bound state of 4.8 MeV in the $(i, j) = (1, 0)$ $N\Xi$ channel and an almost bound state in the $(i, j) = (1, 1)$ $N\Xi$ channel. In this work we will study the strangeness -2 three-baryon $(I)J^P = (3/2)1/2^+$ state using existing $N\Xi$ interactions with attractive isospin 1 channels. Thus, for the present analysis we will follow the line of the Nijmegen $N\Xi$ interaction of Refs. [14, 15] and also the constituent quark cluster model (CQCM) analysis of Ref. [17].

In order to uncover the main features of this system we will perform first a Faddeev calculation where the NN subsystem is in the $(i, j) = (1, 0)$ channel and the $N\Xi$ subsystem is either in the $(i, j) = (1, 1)$ or $(i, j) = (1, 0)$ channel. We use for the NN interaction the Reid soft-core potential [18], and for the $N\Xi$ interaction in any of the two channels the potential,

$$V(r) = -A e^{-\alpha r}/r + B e^{-\beta r}/r, \quad (1)$$

with $A = 332$ MeV fm, $B = 1500$ MeV fm, $\alpha = 1.5$ fm $^{-1}$, and $\beta = 4.0$ fm $^{-1}$. With this interaction the $N\Xi$ subsystem is almost bound and its phase shift changes sign at about the same energy as the NN subsystem.

Since two of the particles of this system are identical fermions, the corresponding Faddeev equations are [11]:

$$T = a t_N^{N\Xi} G_0 T + 2 b t_N^{N\Xi} G_0 t_{\Xi}^{NN} G_0 T, \quad (2)$$

where the subscript (superscript) in the two-body amplitudes t denotes the spectator (interacting pair), and the constants a and b contain the spin-isospin recoupling coefficients and the phase arising from the reduction for identical-particles [11].

For the ΞNN three-baryon system with $J = 3/2$, only the $(i, j) = (1, 1)$ $N\Xi$ channel contributes and one finds $a = -1$, $b = 0$. Thus, due to the negative sign of a , the $N\Xi$ interaction is effectively repulsive and, therefore, no bound state is possible for $J = 3/2$ in spite of the attraction of the $N\Xi$ subsystem. The minus sign in a is a consequence of the identity of the two nucleons since the first term of the r.h.s. of Eq. (2) proceeds through Ξ exchange and it corresponds to a diagram where the initial and final states differ only in that the two identical fermions have been interchanged which brings the minus sign. This effect has been pointed out before [19].

In the case of the ΞNN three-baryon system with $J = 1/2$ one finds: $a = 1/2$, $b = 3/4$ for the $(i, j) = (1, 1)$ $N\Xi$ channel and $a = -1/2$, $b = 1/4$ for the $(1, 0)$ channel. Thus, in the first case both terms in the r.h.s. of Eq. (2) are attractive while in the second case the first term is effectively repulsive and the second term is attractive, but a factor of three smaller than that of the previous case so that effectively the $(i, j) = (1, 0)$ $N\Xi$ channel is weakly attractive and the $(i, j) = (1, 1)$ $N\Xi$ channel is the dominant one.

If we solve Eq. (2) for the ΞNN state $(I)J^P = (3/2)1/2^+$ using as input the NN Reid soft-core potential [18] and only the $(i, j) = (1, 0)$ $N\Xi$ channel, no bound state is obtained. On the other hand, using instead only the $(i, j) = (1, 1)$ $N\Xi$ channel, we obtain a binding energy of 269 keV, what confirms its dominant character. If we now let the $(i, j) = (1, 1)$ $N\Xi$ channel to become bound by increasing the parameter A of the interaction then, as the binding energy of the two-body subsystem increases, the binding energy of the three-body system increases as well. Thus, for example, with $A = 482$ MeV fm, the $(i, j) = (1, 1)$ $N\Xi$ subsystem is bound by 8.3 MeV, similar to the ESC08c Nijmegen model [14], and the three-body system has a binding energy of 6.35 MeV. Similarly, with $A = 399$ MeV fm, the $(i, j) = (1, 1)$ $N\Xi$ subsystem is bound by 1.56 MeV, in agreement with the recent ESC08c Nijmegen model update [15], and the three-body system has a binding energy of 2.50 MeV.

We have finally performed a full-fledged Faddeev calculation [5] using the constituent

quark cluster model analysis of Ref. [17]. Including only the $(i, j) = (1, 1)$ $N\Xi$ channel we get a binding energy of 84 keV, while including both $N\Xi$ channels one gets a binding energy of 429 keV. Notice that the binding energies obtained from this model are much smaller than those obtained from the Nijmegen-inspired model, since here the dominant channel, the $(i, j) = (1, 1)$ $N\Xi$ subsystem, is almost bound while there it has a binding energy of 8.3 MeV [14] or 1.56 MeV [15].

Let us note that current Ξ hypernuclei studies [2–4] have been also performed by means of $N\Xi$ interactions derived from the Nijmegen interaction models and thus our study complements such previous works for the simplest system that could be studied exactly, the ΞNN system. One can also find in the literature models for the baryon-baryon interaction in the strangeness -2 sector based in EFT calculations [20], that also show $i = 1$ $N\Xi$ attraction, although one cannot conclude the strength of the interaction due to the huge effective ranges reported.

To summarize, we have shown that using either simple phenomenological potentials or a full-fledged Faddeev calculation with realistic $N\Xi$ interactions, derived either from the latest Nijmegen models or from a constituent quark cluster model, there may exist a Ξ hypernucleus with baryon number three and quantum numbers $(I)J^P = (3/2)1/2^+$. We have highlighted the particular interest of the $I = 3/2$ channels, because they are decoupled from the $N\Lambda\Lambda$ state, and the $J^P = 1/2^+$ state where the Pauli principle works favorably. Besides, this state would benefit from additional binding coming from the Coulomb potential, that in the case of the Nijmegen inspired models would account for a few MeV. The equivalent $J^P = 3/2^+$ state, with maximum coupling in spin and isospin space, could not be bound in spite of the attraction of both two-body subsystems, due to the phase appearing from the reduction for identical-particles in the Faddeev equations, that make the $N\Xi$ interaction effectively repulsive. This result is a consequence of the expected $N\Xi$ attraction in isospin 1 channels, as it occurs in the ESC08c Nijmegen potential [14, 15] and the CQCM of Ref. [17]. One should emphasize the importance that the $i = 1$ $N\Xi$ attractions are strong, because in the present experimental situation the most promising production of Ξ^- hypernuclei is by (K^-, K^+) reactions. Then, any produced Ξ^- systems have to be in neutron-excess, because of $\Delta i_z = 1$ transfers on available nuclear targets. For such systems, the $i = 1$ $N\Xi$ attraction works favorably.

Let us finally comment on the possible detection of this state. Out of the four isospin

components,

$$\begin{aligned}
|3/2, +3/2\rangle &= pp\Xi^0, \\
|3/2, +1/2\rangle &= \frac{1}{\sqrt{3}} \left[(pn + np)\Xi^0 + pp\Xi^- \right], \\
|3/2, -1/2\rangle &= \frac{1}{\sqrt{3}} \left[(pn + np)\Xi^- + nn\Xi^0 \right], \\
|3/2, -3/2\rangle &= nn\Xi^-,
\end{aligned}$$

the $i_z = -1/2$ component would also benefit from electromagnetic attraction between opposite charge particles without penalizing electromagnetic repulsion between any of the pairs, reenforcing the possible existence of this tribaryon beyond the attractive nuclear contribution. In brief, this new type of element, if it does exist, would provide a great opportunity for extending our knowledge to some unreached part in our matter world.

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